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9-1 Study Guide and Intervention

Section Title

The **probability** of a simple event is a ratio that compares the number of favorable outcomes to the number of possible outcomes. Outcomes occur at **random** if each outcome occurs by chance.

Two events that are the only ones that can possibly happen are **complementary events**. The sum of the probabilities of complementary events is 1.

Example 1 **What is the probability of rolling a multiple of 3 on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces.**

$$P(\text{multiple of 3}) = \frac{\text{multiples of 3 possible}}{\text{total numbers possible}}$$

$= \frac{2}{6}$ Two numbers are multiples of 3: 3 and 6.

$= \frac{1}{3}$ Simplify.

The probability of rolling a multiple of 3 is $\frac{1}{3}$ or about 33.3%.

Example 2 **What is the probability of *not* rolling a multiple of 3 on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces?**

$P(A) + P(\text{not } A) = 1$

$\frac{1}{3} + P(\text{not } A) = 1$ Substitute $\frac{1}{3}$ for $P(A)$.

$-\frac{1}{3} \qquad\qquad -\frac{1}{3}$ Subtract $\frac{1}{3}$ from each side

$P(\text{not } A) = \frac{2}{3}$ Simplify.

The probability of *not* rolling a multiple of 3 is $\frac{2}{3}$ or about 66.7%.

Exercises

A set of 30 cards is numbered 1, 2, 3, ..., 30. Suppose you pick a card at random without looking. Find the probability of each event. Write as a fraction in simplest form.

1. $P(12)$

2. $P(2 \text{ or } 3)$

3. $P(\text{odd number})$

4. $P(\text{a multiple of 5})$

5. $P(\text{not a multiple of 5})$

6. $P(\text{less than or equal to 10})$

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9-2 Study Guide and Intervention

Sample Spaces

A game in which players of equal skill have an equal chance of winning is a **fair game**. A **tree diagram** or table is used to show all of the possible outcomes, or **sample space**, in a probability experiment.

Example 1 WATCHES **A certain type of watch comes in brown or black and in a small or large size. Find the number of color-size combinations that are possible.**

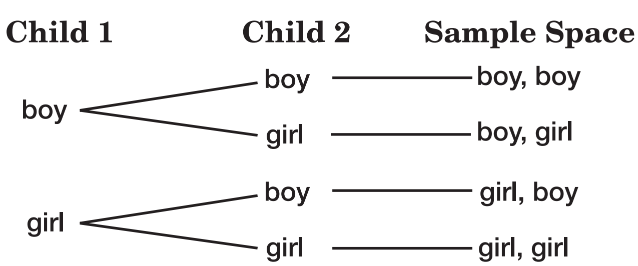
Make a table to show the sample space. Then give the total number of outcomes.

Color	Size
Brown	Small
Brown	Large
Black	Small
Black	Large

There are four different color and size combinations.

Example 2 CHILDREN **The chance of having either a boy or a girl is 50%. What is the probability of the Smiths having two girls?**

Make a tree diagram to show the sample space. Then find the probability of having two girls.



The sample space contains 4 possible outcomes. Only 1 outcome has both children being girls. So, the probability of having two girls is $\frac{1}{4}$.

Exercises

For each situation, make a tree diagram or table to show the sample space. Then give the total number of outcomes.

1. choosing an outfit from a green shirt, blue shirt, or a red shirt, and black pants or blue pants

2. choosing a vowel from the word COUNTING and a consonant from the word PRIME

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The Fundamental Counting Principle

If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then the event *M* followed by *N* can occur in $m \times n$ ways. This is called the **Fundamental Counting Principle**.

Example 1 CLOTHING **Andy has 5 shirts, 3 pairs of pants, and 6 pairs of socks. How many different outfits can Andy choose with a shirt, pair of pants, and pair of socks?**

$$\underbrace{5}_{\text{number of shirts}} \cdot \underbrace{3}_{\text{number of pants}} \cdot \underbrace{6}_{\text{number of socks}} = \underbrace{90}_{\text{total number of outfits}}$$

Andy can choose 90 different outfits.

Exercises

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

1. rolling two number cubes

2. tossing 3 coins

3. picking one consonant and one vowel

4. choosing one of 3 processor speeds, 2 sizes of memory, and 4 sizes of hard drive

5. choosing a 4-, 6-, or 8-cylinder engine and 2- or 4-wheel drive

6. rolling 2 number cubes and tossing 2 coins

7. choosing a color from 4 colors and a number from 4 to 10

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9-4 Study Guide and Intervention

Permutations

The expression *n* **factorial** (*n*!) is the product of all counting numbers beginning with *n* and counting backward to 1. A **permutation** is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.

Example 1 **Find the value of 5!.**

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial
$= 120$ Simplify.

Example 2 **Find the value of 4! · 2!.**

$4! \cdot 2! = 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1$ Definition of factorial
$= 48$ Simplify.

Example 3 BOOKS **How many ways can 4 different books be arranged on a bookshelf?**

This is a permutation that can be written as 4!. Suppose the books are placed on the shelf from left to right.

There are **4** choices for the first book.
There are **3** choices that remain for the second book.
There are **2** choices that remain for the third book.
There is **1** choice that remains for the fourth book.

$4! = 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial
$= 24$ Simplify.

So, there are 24 ways to arrange 4 different books on a bookshelf.

Exercises

Find the value of each expression.

1. 3!

2. seven factorial

3. $6! \cdot 3!$

4. $9 \cdot 8 \cdot 7$

5. How many ways can you arrange the letters in the word GROUP?

6. How many different 4-digit numbers can be created if no digit can be repeated? Remember, a number cannot begin with 0.

9-5 Study Guide and Intervention

Combinations

An arrangement, or listing, of objects in which order is *not* important is called a **combination**. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

Example 1 **Jill was asked by her teacher to choose 3 topics from the 8 topics given to her. How many different three-topic groups could she choose?**

There are $8 \cdot 7 \cdot 6$ permutations of three-topic groups chosen from eight. There are 3! ways to arrange the groups.

$$\frac{8 \cdot 7 \cdot 6}{3!} = \frac{336}{6} = 56$$

So, there are 56 different three-topic groups.

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

Example 2 **On a quiz, you are allowed to answer any 4 out of the 6 questions. How many ways can you choose the questions?**

This is a combination because the order of the 4 questions is not important. So, there are $6 \cdot 5 \cdot 4 \cdot 3$ permutations of four questions chosen from six. There are 4! or $4 \cdot 3 \cdot 2 \cdot 1$ orders in which these questions can be chosen.

$$\frac{6 \cdot 5 \cdot 4 \cdot 3}{4!} = \frac{360}{24} = 15$$

So, there are 15 ways to choose the questions.

Example 3 **Five different cars enter a parking lot with only 3 empty spaces. How many ways can these spaces be filled?**

This is a permutation because each arrangement of the same 3 cars counts as a distinct arrrangement. So, there are $5 \cdot 4 \cdot 3$ or 60 ways the spaces can be filled.

Exercises

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

1. How many ways can 4 people be chosen from a group of 11?

2. How many ways can 3 people sit in 4 chairs?

3. How many ways can 2 goldfish be chosen from a tank containing 15 goldfish?

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Problem-Solving Investigation: Act It Out

By acting out a problem, you are able to see all possible solutions to the problem being posed.

Example CLOTHING **Ricardo has two shirts and three pairs of pants to choose from for his outfit to wear on the first day of school. How many different outfits can he make by wearing one shirt and one pair of pants?**

Understand We know that he has two shirts and three pairs of pants to choose from. We can use a coin for the shirts and an equally divided spinner labeled for the pants.

Plan Let's make a list showing all possible outcomes of tossing a coin and then spinning a spinner.

Solve H = Heads
T = Tails
Spinner = 1, 2, 3

Flip a Coin	Spin a Spinner
H	1
H	2
H	3
T	1
T	2
T	3

There are six possible outcomes of tossing a coin and spinning a spinner. So, there are 6 different possible outfits that Ricardo can wear for the first day of school.

Check Tossing a coin has two outcomes and there are two shirts. Spinning a three-section spinner has three outcomes and there are three pairs of pants. Therefore, the solution of 6 different outcomes with a coin and spinner represent the 6 possible outfit outcomes for Ricardo.

Exercises

1. SCIENCE FAIR There are 4 students with projects to present at the school science fair. How many different ways can these 4 projects be displayed on four tables in a row?

2. GENDER Determine whether tossing a coin is a good way to predict the gender of the next 5 babies born at General Hospital. Justify your answer.

3. OLYMPICS Four runners are entered in the first hurdles heat of twelve heats at the Olympics. The first two move on to the next round. Assuming no ties, how many different ways can the four runners come in first and second place?

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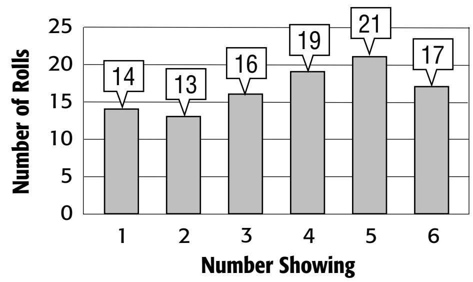
Theoretical and Experimental Probability

Experimental probability is found using frequencies obtained in an experiment or game. **Theoretical probability** is the expected probability of an event occurring.

Example 1 **The graph shows the results of an experiment in which a number cube was rolled 100 times. Find the experimental probability of rolling a 3 for this experiment.**



$$P(3) = \frac{\text{number of times 3 occurs}}{\text{number of possible outcomes}}$$

$$= \frac{16}{100} \text{ or } \frac{4}{25}$$

The experimental probability of rolling a 3 is $\frac{4}{25}$, which is close to its theoretical probability of $\frac{1}{6}$.

Example 2 **In a telephone poll, 225 people were asked for whom they planned to vote in the race for mayor. What is the experimental probability of Juarez being elected?**

Candidate	Number of People
Juarez	75
Davis	67
Abramson	83

Of the 225 people polled, 75 planned to vote for Juarez.

So, the experimental probability is $\frac{75}{225}$ or $\frac{1}{3}$.

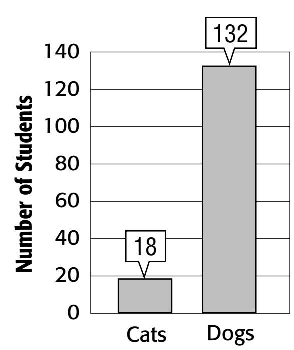
Example 3 **Suppose 5,700 people vote in the election. How many can be expected to vote for Juarez?**

$\frac{1}{3} \cdot 5{,}700 = 1{,}900$

About 1,900 will vote for Juarez.

Exercises

For Exercises 1–3, use the graph of a survey of 150 students asked whether they prefer cats or dogs.



1. What is the probability of a student preferring dogs?

2. Suppose 100 students were surveyed. How many can be expected to prefer dogs?

3. Suppose 300 students were surveyed. How many can be expected to prefer cats?

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9-8 Study Guide and Intervention

Compound Events

A **compound event** consists of two or more simple events. If the outcome of one event does not affect the outcome of a second event, the events are called **independent events**. The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

Example 1 **A coin is tossed and a number cube is rolled. Find the probability of tossing tails and rolling a 5.**

$P(\text{tails}) = \frac{1}{2}$ $\quad P(5) = \frac{1}{6}$

$P(\text{tails and } 5) = \frac{1}{2} \cdot \frac{1}{6} \text{ or } \frac{1}{12}$

So, the probability of tossing tails and rolling a 5 is $\frac{1}{12}$.

Example 2 MARBLES **A bag contains 7 blue, 3 green, and 3 red marbles. If Agnes randomly draws two marbles from the bag, replacing the first before drawing the second, what is the probability of drawing a green and then a blue marble?**

$P(\text{green}) = \frac{3}{13}$ 13 marbles, 3 are green

$P(\text{blue}) = \frac{7}{13}$ 13 marbles, 7 are blue

$P(\text{green, then blue}) = \frac{3}{13} \cdot \frac{7}{13} = \frac{21}{169}$

So, the probability that Agnes will draw a green, then a blue marble is $\frac{21}{169}$.

Exercises

1. Find the probability of rolling a 2 and then an even number on two consecutive rolls of a number cube.

2. A penny and a dime are tossed. What is the probability that the penny lands on heads and the dime lands on tails?

3. Lazlo's sock drawer contains 8 blue and 5 black socks. If he randomly pulls out one sock, what is the probability that he picks a blue sock?

Lesson 9-8